Faculty of Science, Technology, Engineering and Mathematics
MST210 Mathematical methods, models and modelling

MST210

TMA 02 2019J

Covers Units 1, 2 and 3 **Cut-off date 20 November 2019**

The module website gives details of how to submit assignments for this module.

The marks allocated to each part of each question are indicated in brackets in the margin.

You may use Maxima to check answers, but it should not be used otherwise to answer TMA questions except where explicitly indicated. TMA answers must show full working to attract full marks.

In order to encourage you to present your solutions to the TMA questions in a good mathematical style, your tutor will comment on how you:

- use correct mathematical notation
- define any symbols that you introduce in formulating and solving a problem
- give references for standard formulas and derivations
- include comments and explanations within your mathematics
- explicitly state results and conclusions, giving answers to an appropriate degree of accuracy and interpreting answers in the context of the question
- draw diagrams and graphs.

These features are seen as being essential to complementing your mathematical skills. Your tutor will make comments on how well you achieve these objectives and give you guidance on how to satisfy them.

Five of the marks for this TMA will be allocated to the way you write your solutions. It is expected that most students will receive the majority of these presentation marks; such marks are included in TMAs to encourage, and emphasise the need for, thinking about how you present your mathematics.

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Question 1 (Unit 1) – 15 marks

Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 15y = -5x - 16e^{-x}. \qquad [15]$$

Question 2 (Unit 2) – 4 marks

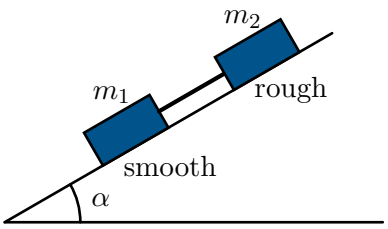
The points A and B have coordinates $(3, -1, 2)$ and $(0, 1, 2)$, respectively, with respect to a three-dimensional Cartesian coordinate system with origin O. The vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are denoted by $\mathbf{a}$ and $\mathbf{b}$, respectively.

In this question you should express all vectors in the form $a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are Cartesian unit vectors.

Calculate the cross product of $\mathbf{a}$ and $\mathbf{b}$. [4]

Question 3 (Unit 2) – 19 marks

Two blocks, of masses m_1 and m_2, are connected by a model string, as shown below. Both blocks lie on a plane that is inclined to the horizontal at an angle α. The part of the plane supporting mass m_1 is smooth, so that there is no frictional force. The upper part of the plane is rough, and the coefficient of static friction between the block of mass m_2 and the plane is μ. The system is in equilibrium, and the string is taut.



This question follows Procedure 2 for solving statics problems, on page 129 of Unit 2, and the steps of this procedure appear in the parts of the question.

(a) ◀ Choose axes ▶

Draw a diagram, marking clearly your choice of coordinate axes. [1]

(b) ◀ Draw force diagrams ▶

Modelling the blocks as particles, draw two force diagrams showing all the forces acting on the two particles. [4]

(c) ◀ Apply law(s) ▶

Apply appropriate laws to find two vector equations, one scalar equation and one inequality. [4]

(d) ◀ Solve equation(s) ▶

Assuming that the system remains in equilibrium, show that

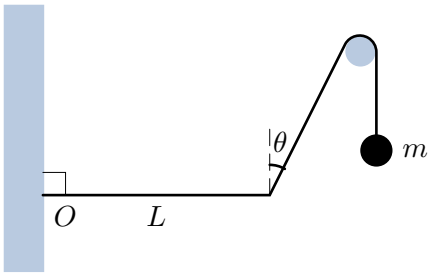
$$(m_1 + m_2)\tan\alpha \leq \mu m_2. \qquad [8]$$

(e) ◀ Interpret solution ▶

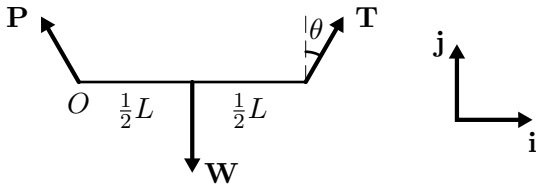
If the mass of the first block is $m_1 = 1\,\text{kg}$, the coefficient of friction for the second block is $\mu = \frac{3}{4}$ and $\tan\alpha = \frac{1}{3}$, for what range of m_2 will the system remain in equilibrium? [2]

Question 4 (Unit 2) – 7 marks

A heavy pole of mass $2M$ and length L is freely hinged to a vertical wall at a point O. The weight of the pole is supported by a wire attached to the end, which goes over a pulley and is connected to a particle of mass m. The wire makes an angle θ with the upward vertical direction, as shown in the following diagram.



Model the wire as a model string and the pulley as a model pulley. Three forces act on the pole: the weight of the pole **W**, the tension force due to the wire **T**, and a force **P** due to the hinge. These forces acting on the pole are shown in the force diagram below. Choose horizontal and vertical unit vectors and an origin at O, as shown in the diagram.



Assume that the pole is in equilibrium. By taking torques about O, derive an equation that gives the angle θ in terms of the parameters given above. [7]

Question 5 (Unit 3) – 16 marks

A ball of mass m is thrown vertically upwards with initial speed u, and travels upwards under the influence of gravity and air resistance. Use the quadratic model of air resistance with the ball modelled as a sphere of effective diameter D.

This question follows Procedure 1 on page 197 of Unit 3, and the steps of this procedure appear in the parts of the question.

(a) ◀ Draw picture ▶

Draw a picture and mark on it any relevant information. [*1*]

(b) ◀ Choose axes ▶

Choose an axis for this problem. [*1*]

(c) ◀ State assumptions ▶

State any assumptions made. [*1*]

(d) ◀ Draw force diagram ▶

Draw a force diagram. [*1*]

(e) ◀ Apply Newton's second law ▶

Apply Newton's second law to obtain the equation

$$\frac{dv}{dt} = -k\Big(v^2 + \frac{g}{k}\Big),$$

where v is the speed of the ball, t is the time elapsed since the ball was released, g is the magnitude of the acceleration due to gravity, and $k = c_2D^2/m$ is a constant. [*3*]

(f) ◀ Solve differential equation ▶

Solve the differential equation and apply the initial condition to find the time t in terms of the speed v and the constants given above. [*6*]

(g) ◀ Interpret solution ▶

Use your equation to show that the time taken to reach the maximum height attained by the ball, $t_{\max}$, is given by

$$t_{\max} = \frac{1}{\sqrt{gk}} \arctan\left(\sqrt{\frac{k}{g}}\, u\right).$$

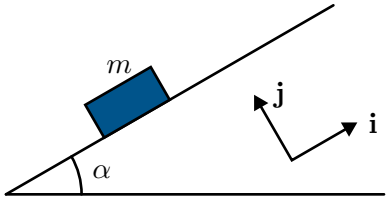
This model predicts that as the initial speed u increases, there is an upper limit for the time to reach maximum height. Calculate this upper limit for a beach ball with diameter 0.4 metres and mass 0.2 kilograms (use $c_2 = 0.2$ and $g = 9.81$). [*3*]

Question 6 (Unit 3) – 9 marks

A block of mass m is moving on a rough slope with coefficient of sliding friction μ'. Initially the block is moving up the slope with speed u.

(a) Draw a force diagram showing all the forces acting on the block. [*1*]

(b) Choose axes with **i** pointing up the slope and **j** perpendicular to the slope, as shown in the following diagram.



Write the forces acting on the block in terms of the unit vectors **i** and **j**. [*2*]

(c) When Newton's second law is applied to this mechanical system, it is found that the acceleration of the block up the slope, a, is given by $a = -g(\sin\alpha + \mu'\cos\alpha)$. Use this result to show that the distance x travelled by the block until it comes to rest is given by

$$x = \frac{u^2}{2g(\sin\alpha + \mu'\cos\alpha)}.$$ [*3*]

(d) This result is to be used in the design of a ramp that will slow down blocks that enter at a fixed speed. You are given the following data.

Length of ramp, x	$0.85\,\text{m}$
Initial speed, u	$4\,\text{m}\,\text{s}^{-1}$
Acceleration due to gravity, g	$9.81\,\text{m}\,\text{s}^{-2}$
Coefficient of sliding friction, μ'	$\frac{5}{12}$

Using Maxima, or otherwise, find to the nearest degree the angle α that the slope needs to have to slow the block to rest at the top of the slope. Include in your answer your input to Maxima, the output that Maxima produced, and your interpretation of this output. [*3*]

(*Hint*: Use the Maxima command `find_root` to solve the equation in part (c). As an example of usage, the Maxima expression `find_root(x^2=2,x,1,2)` will find the value of x between 1 and 2 for which $x^2 = 2$; that is, it will return an approximation to $\sqrt{2}$.)